



Mathematics Framework

Together, we wondered what it is to engage in the inspired practices of teaching and learning mathematics.

January 2020

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Letter from the Mathematics Task Force

Dear Colleagues:

Lakewood City Schools values education as the great equalizer. From our preschool students to our graduates, we strive to instill a love for mathematics and create mathematically literate environments where all have the opportunity to learn, grow and achieve. With this goal in mind, teachers and administrators throughout the district collaborated over a two-year period to create a comprehensive mathematics framework. As we work toward fulfilling Lakewood's Vision of a Graduate, we believe this document will serve as a helpful guide for our educators to understand what mathematics education means to Lakewood City Schools.

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Beliefs

Mathematics invests humans in understanding, appreciating, and experiencing their world.

While mathematics success is a fundamental component of the Lakewood City Schools' Vision of a Graduate, it is essential that mathematics instruction be based on Ohio standards and that district curriculum allows for teacher responsiveness to the needs of students. The following six points are Lakewood City School District's (PK-16) vision for mathematics instruction.

A Lakewood Student will experience:



- Mathematics integrated into the world around them.



- Opportunities to communicate their ideas and results effectively and creatively.



- Ambitious expectations, with accommodation and differentiation.
- A culture where they can confidently engage in complex mathematical tasks.
- A community where teachers help them make, refine, and explore questions using evidence, and a variety of reasoning and proof techniques to confirm or disprove their results (modeling).



- A rich curriculum, where there are opportunities to learn important mathematical concepts and procedures.
- Opportunities for mathematical recreation to explore, apply, construct and enjoy mathematics.



- Knowledgeable teachers who are continually growing as professionals.
- Environments where they will work effectively with others.



- An environment where flexibility, resourcefulness, making mistakes, and solving problems is encouraged.

Important Links

Ohio's Math Standards

[Early Learning & Development - Cognition and General Knowledge \(Pre School\)](#)

[Ohio's Learning Standards for Mathematics](#)

[Ohio's Math Model Curricula](#)

Lakewood Literacy Framework

[Lakewood's Literacy Framework](#)

Standards for Mathematical Practice

The Standards for Mathematical Practice (SMP) are the foundation for mathematical thinking and practice for students K-12, as well as guidance for teachers in developing more advanced mathematical understandings. Think of these standards as a guide to creating more complex and absorbing learning experiences, that can be applied to everyday life, instead of being left in the classroom.

1. Make sense of problems and persevere in solving them.

Mathematically proficient students must understand the problem, figure out how to solve it, and then work until it is finished. While students work through more difficult problems, they focus on the process of solving the problem instead of just getting to the correct answer.

2. Reason abstractly and quantitatively.

Mathematically proficient students understand there are multiple ways to break apart the problem in order to find the solution. Using symbols, pictures or other representations to describe the different sections of the problem will allow students to use context skills rather than standard algorithms.

3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students use common mathematical language to discuss and explain math as well as support or object others' work. Math vocabulary is integrated in order for students to be able to communicate effectively. Prompting students to participate further in class mathematical discussion will help build student communication skills.



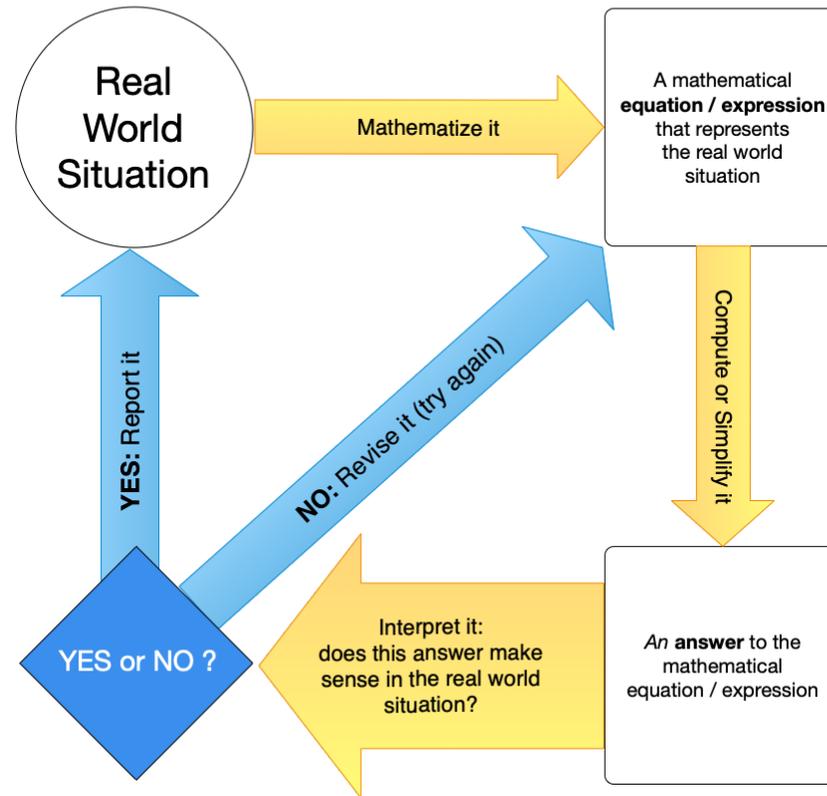
CAUTION: Overuse of individual instructional technology can limit valuable mathematical discourse.

4. Model with mathematics

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. Learning to model with mathematics means that students will use math skills to problem-solve real world situations. Using real world situations to show how math can be used in many different aspects of life helps math to be relevant outside of math class. Math doesn't end at the classroom door.

Modeling links classroom mathematics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics to analyze real-life situations, to understand them better, and to make good decisions.

The Mathematical Modeling Cycle



CAUTION: Although the use of manipulatives makes an abstract concept concrete, the use of manipulatives alone does not constitute mathematical modeling. There must be a real world situation in order for mathematical modeling to take place. If we tell students the information they need to solve the problem upfront...it's not modeling. Students must take the lead on identifying variables and mathematizing real world situations.

[Ohio High School Modeling Standard \(see pp 59-60\)](#) — a good description of the modeling cycle that is also useful for middle and elementary teachers.

5. Use appropriate tools strategically

Mathematically proficient students consider the available tools when solving a mathematical problem. In order for students to learn what tools should be used in problem solving it is important to remember that no one will be guiding students through the real world – telling them which mathematics tool to use. When engaged in open ended problems students can select which math tools to use and discuss what worked and what didn't.

6. Attend to precision

Mathematically proficient students try to communicate precisely with others. They work to use clear definitions in discussion with others and in their own reasoning. They calculate accurately, efficiently, and express numerical answers with a degree of precision appropriate for the problem context.

7. Look for and make use of structure

Mathematically proficient students can identify different strategies for problem solving. They identify similar patterns, use repeated reasoning, and multiple tools in mathematics to solve more complex problems by breaking them into separate parts.

8. Look for and express regularity in repeated reasoning

Mathematically proficient students understand the broader application of patterns and see the structure in similar situations by solving one problem the way it was taught as well as replicating that problem-solving technique as appropriate with other problems.

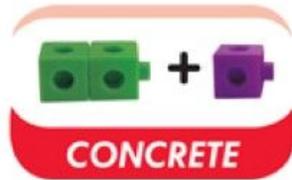
[Standards For Mathematical Practices Look-for-Tool](#)--describes possible evidence of Standards for Mathematical Practices for students/teachers.

[Standards for Mathematical Practices Progression through Grade Levels](#)--describes expectations for each standard by grade level (K-12).

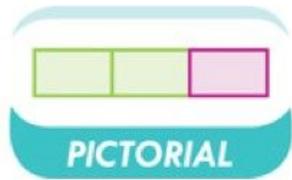
[Questions to develop mathematical thinking and the Standards for Mathematical Practices](#)--question stems to prompt application of the SMP.

Developing a Foundation in Mathematics

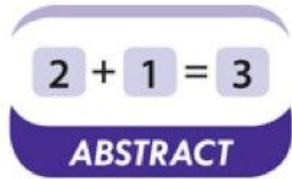
Concrete-Pictorial-Abstract Learning Progression – A Classical Approach to Laying a Strong Foundation



Concepts are introduced through hands-on experiences with manipulatives, the necessary first step for young children who think in concrete terms. Even older children may revert to this mode when learning a new skill.



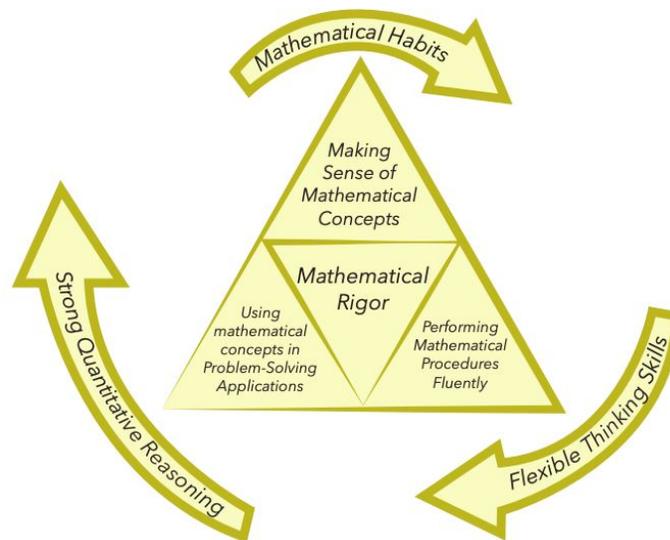
Students visualize the concept and represent it pictorially through models like number bonds and bar models, which represents the “mental image” mode. This mode of thinking assists students in dissecting challenging word problems and developing logical thinking.



Students only use abstract numbers and symbols when they have enough context to understand what they mean, i.e. true abstract thinking without using concrete objects or pictures, a skill required by algebra, trigonometry and calculus later on.

From <https://www.maryvaleacademy.ca/academics/math/>

Understanding Mathematical Rigor



MATHEMATICAL RIGOR: Rigor doesn't just mean "harder" or "more difficult." Rigor in math teaching means focusing with equal intensity on students' conceptual understanding, procedural fluency, and **ability to apply** what they know to real-world, problem-solving situations.

CONCEPTUAL UNDERSTANDING: Making sense of the math; Reasoning about and understanding math concepts and ideas; Using concrete, pictorial, abstract models to build an understanding of mathematical concepts.

PROCEDURAL FLUENCY: Doing the math; Building an understanding of mathematical concepts. Developing knowledge of procedures and strategies. Developing an understanding of how and when to flexibly, accurately, and efficiently use various procedures and strategies. The more energy students use for procedures, the less energy available for problem-solving. The less energy for problem-solving, the less likely students are to gain additional conceptual understandings. A lack of procedural fluency contributes to a lack of conceptual understanding.

APPLICATION: Using the math; Developing the capacity to solve a wide range of problems in various contexts by reasoning, thinking, and applying the mathematics learned to real-world, problem-solving situations.

The connective tissue of a coherent approach to math instruction includes: mathematical habits, flexible thinking skills, and strong quantitative reasoning.

Wondering if the tasks your students are engaged in are mathematically rigorous? Check out this link. [Identifying Rigorous Tasks](#)

Differentiation

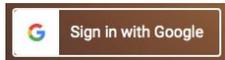
As we provide inclusive math education, on-going formative assessments help to determine groupings for differentiation. Additionally, summative assessments help to inform instruction.

Intervention	Enrichment
<p>The first level of intervention should occur in the classroom as a part of core instruction. Teachers are encouraged to intervene using various options:</p> <ul style="list-style-type: none"> ● Small group instruction within the classroom (stations/centers) ● Reinforce Concrete-Pictorial-Abstract progression ● Tactile Learning /Hands on/Manipulatives ● Using technology ● Multiple times to practice ● Variety of strategies to assist students ● Focusing on the progression of skills - what skills are they missing that can help them access the curriculum <p>Elementary intervention groups are based on diagnostic and teacher input. Components of intervention groups: intentionally group students based on skill, direct instruction time, ongoing monitoring, pre and post testing.</p> <p>Happy Numbers is used to provide skill-based individual instruction for grades K-2.</p> <p>i-Ready can be used to establish personal pathways for elementary grades 3-5 and middle school students.</p> <p>High School intervention courses are offered based on assessments that identify the areas where support is needed.</p> <p>Intervention Central: Response to Intervention RTI RTI Resources Elementary Standard Protocol- 2019- Introduction of Math</p>	<p>The first level of enrichment should occur in the classroom as a part of core instruction. Teachers are encouraged to enrich using various options:</p> <ul style="list-style-type: none"> ● Small group instruction within the classroom (stations/centers) ● Tactile Learning /Hands-on ● Using technology ● Variety of strategies to assist students ● Independent studies or project-based learning that engage higher-level thinking ● Sharing different perspectives and methods with others ● Provide open-ended learning opportunities <p>Elementary Schools offer cluster grouping for mathematics based on gifted identification.</p> <ul style="list-style-type: none"> ● Great Minds Eureka Strategies for Challenging Advanced Learners Webinar <p>Happy Numbers is used to provide skill-based individual instruction for grades K-2.</p> <p>i-Ready can be used to establish personal pathways for elementary grades 3-5 and middle school students.</p> <p>Middle School Advanced courses are offered based on multi-criteria review and may be available for high school credit.</p> <p>High School Advanced and AP courses are offered based on student achievements and interest.</p>

Strategy Bank

Looking for strategies to ramp up engagement during math,
or grow mathematical understandings?
Check out the ideas below.

[Externally Curated Instructional Strategies](#)



*As you open links in this resource you will be prompted to login to Discovery Education. Scroll down and click “Sign in with Google” to obtain access using your Lakewood City Schools account.

[Staff Curated Instructional Strategies](#)

Growing Mathematicians: A PreK-12 Progression

GRADE BAND: PK - K

RESEARCH LINKS

- [Achieve the Core](#)
- [Kindergarten District Adopted Math Curriculum Research](#)

RESEARCH LINKS

- [Marilyn Burns' Blog](#)

GENERAL LINKS

- [Grade Level Folders](#)
- [Great Minds](#) - Digital Access to Eureka Curriculum
- [Engage NY Math](#)
- [K-8 Math Learning Progressions](#)

GENERAL LINKS

- [Coherence Map](#)
- [Embarc](#) - Eureka resources (Topic assessments, flip charts, etc)
- [Achieve the Core Major Cluster Alignment Document](#)
- [Children's literature with math embedded](#)

ESSENTIAL UNDERSTANDINGS

What is necessary for students to know and be able to do?

- Know number names and count sequences
- Count to tell the number of objects
- Compare numbers
- Understand addition as putting together and adding to
- Understand subtraction as taking apart and taking from
- Work with numbers 11-19 to gain foundations for place value

COMPONENTS

What are the necessary components of high-quality instruction? How do we get students there?

- Hands on activities (manipulatives)
- Small groups
- Differentiation & Intervention
- Math games
- Technology
- Collaboration/Speaking/Listening about Math
- Explicitly Teaching of Math
- Authentic Experiences
- Math practice

ASSESSMENT FOR LEARNING

How will we know when students have it?

- Anecdotal notes: observation & checklists
- Common Assessments ([Mid/End of module Eureka](#))
- [Happy Numbers](#) for K

What we will do for students who have it?

- [Small group enrichment tasks.](#)
- Higher levels of Blooms
- Tasks with collaboration ([YouCubed](#), [Illustrative](#), [NRICH](#),)
- [Math talks](#)
- Open ended opportunities ([Open Middle](#), [Robert Kaplinsky](#), [3 Act Tasks](#))

GRADE BAND: PK-K

What are the necessary components of high-quality instruction? How do we get students there? Continued...

- Moving from concrete to abstract (Manipulatives, icons, abstract numbers)
- Repeated practice
- Exploration
- Strategic centers (standards-based) for front loading and skill reinforcement
- Math Talks
- Fluency: Understanding of 3 at PK
- Fluency: Understanding of 5 at K
- PK uses Creative Curriculum
- K uses Eureka

What we will do for students who have it?
Continued...

- Delve deeper into grade level standards
- Peer Teaching

What will we do for students who do not have it?

- Centers: Foundational skills ([Number Sense](#), independent work, front loading)
- [Small group tasks](#)
- Reteach
- Scaffolding
- Differentiation & Intervention
- Use of [Math Tools](#)

GRADE BAND: 1-2

RESEARCH LINKS

- [Grade 1 District Adopted Math Curriculum Research](#)
- [Grade 2 District Adopted Math Curriculum Research](#)

RESEARCH LINKS

- [Marilyn Burns' Blog](#)
- [Achieve the Core](#)

GENERAL LINKS

- [Grade Level Folders](#)
- [Great Minds](#) - Digital Access to Eureka Curriculum
- [Engage NY Math](#)
- [K-8 Math Learning Progressions](#)

GENERAL LINKS

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- [Children's literature with math embedded](#)

ESSENTIAL UNDERSTANDINGS

COMPONENTS

ASSESSMENT FOR LEARNING

What is necessary for students to know and be able to do?

- Mastering Fact [Fluency](#) (0-20)
- Growing Number Sense
- Understanding Place Value
- Solving Word Problems
- Describing shapes and their attributes
- Creating equal groups
- Understanding repeated addition as the foundation for multiplication
- Composing and decomposing numbers
- Using a variety of strategies to add and subtract numbers

What are the necessary components of high-quality instruction? How do we get students there?

- Hands on activities (manipulatives)
- Small group instruction
- Technology
- Teaching curriculum with fidelity
- Games
- [Math Talks](#)
- [Number Sense](#)
- Math Fact [Fluency](#)
- Differentiation & Intervention

How will we know when students have it?

- Common Eureka Assessments
 - [Grade 1](#)
 - [Grade 2](#)
- Exit tickets
- Observation
- [Zearn/Happy Numbers](#) Data
- [Formative Loop](#), Eureka Sprints, [District Fluency Folder](#) - assessment of fact fluency

What we will do for those that have it?

- [Small group enrichment](#)
- Higher levels of [Blooms](#)
- Tasks with collaboration ([YouCubed](#), [Illustrative](#))
- Open ended opportunities-[Open Middle](#), [3 Act Tasks](#), [Robert Kaplinsky](#)

GRADE BAND: 1-2

What we will do for those that have it?

Continued...

- Delve deeper into grade level standards
- Peer Teaching
- Centers: [Problem of the Month/Week](#)
- [Math Talks](#)
- [Links for Enrichment](#)

What will we do for those who do not have it?

- [Centers](#)
- Small group
- Reteach
- Scaffolding
- Differentiation & Intervention
- Use of [Math Tools](#)
- [Number Sense](#)

GRADE BAND: 3-5

RESEARCH LINKS

- [Grade 3 District Adopted Math Curriculum Research](#)
- [Grade 4 District Adopted Math Curriculum Research](#)
- [Grade 5 District Adopted Math Curriculum Research](#)

RESEARCH LINKS

- [Marilyn Burns' Blog](#)
- [Achieve the Core](#)

GENERAL LINKS

- [Grade Level Folders](#)
- [Great Minds](#) - Digital Access to Eureka Curriculum
- [Engage NY Math](#)
- [K-8 Math Learning Progressions](#)

GENERAL LINKS

- [Coherence Map](#)
- [Embarc](#) - Eureka resources (Topic assessments, flip charts, etc)
- [Achieve the Core Major Cluster Alignment Document](#)
- [Children's literature with math embedded](#)
- [Greg Tang Math](#)
- [Released AIR Items](#)

ESSENTIAL UNDERSTANDINGS

COMPONENTS

ASSESSMENT FOR LEARNING

What is necessary for students to know and be able to do?

- Mastering Fact [Fluency](#)
- Understanding of base 10 number system
- Renaming units, composing and decomposing numbers
- Utilizing Mathematical Practices
- Using the four operations to solve word problems (whole numbers, decimals & fractions)
- Understanding and manipulating fractions and apply whole number concepts to fractions

What are the necessary components of high-quality instruction? How do we get students there?

- Use of small group instruction
- Student collaboration
- Using frequent assessment
- Incorporation of technology
- Flexibility in groupings, whole class, peer work, teacher time, 1:1
- Growing student vocabulary and the language of mathematics
- Encouraging struggle with error analysis ([Growth Mindset](#))

How will we know when students have it?

- Eureka Summative Assessments
 - [Grade 3](#)
 - [Grade 4](#)
 - [Grade 5](#)
- Exit Tickets
- Teacher Observations
- [Released test items \(AIR, PARCC, Smarter Balance...\)](#)
- Eureka Sprints/[Formative Loop](#), [District Fluency Folder](#)-assessment of fact fluency
- [iReady](#)

GRADE BAND: 3-5

What is necessary for students to know and be able to do? Continued...

- Engaging in high-level talk about math
- Writing equations based on word problems
- Arriving at reasonable answers and estimations, adjusting numbers, demonstrating flexibility with numbers
- Understanding and applying concepts that underlie standard algorithms across whole numbers, decimals and fractions (borrowing, regrouping, renaming, bundling, unbundling)
- Understanding the meaning of multiplication, division, addition, subtraction
- Conceptual understanding of area & volume

What are the necessary components of high-quality instruction? How do we get students there? Continued...

- Conversation ([Number Talks](#)), rich discussions, conceptual development
- Use of Manipulatives GCCTM, Mathematical Mindsets

What we will do for those that have it?

- [Providing enrichment tasks](#)
- Consolidating lessons/pacing
- Self-directed learning ([Freckle](#), [iReady](#), [Khan](#))
- MathMenus, [Problem of the Month/Week](#)
- [Gifted Math Resources](#)
- [NRICH](#)
- [Open Middle](#)
- Math Tasks ([YouCubed](#), [Illustrative](#), [Robert Kaplinsky](#), [3 Act Tasks](#))
- [Number Sense](#)

What will we do for those who do not have it?

- Small group
- Reteach
- Manipulatives
- Technology ([Zearn](#) & [Zearn](#) printables, [Freckle self guided math](#))
- Backteach Eureka and foundational standards from prior grades
- After School Math Support

GRADE BAND: 6-12

RESEARCH LINKS

- [Grade 6 District Adopted Math Curriculum Research](#)
- [Grade 7 District Adopted Math Curriculum Research](#)
- [Grade 8 District Adopted Math Curriculum Research](#)
- [High School District Adopted Math Curriculum Research](#)

RESEARCH LINKS

- [Marilyn Burns' Blog](#)
- [Achieve the Core](#)
- [iReady](#)
- [You Cubed - Dr. Jo Boaler](#)

GENERAL LINKS

- [Discovery Math Techbook](#) (6-8 Curriculum)
- [Pearson enVision A-G-A Textbook](#) (Algebra 1, Geometry, Algebra 2 Curriculum)
- [Learning Progressions](#)
- [Coherence Map](#)
- [Free Math Worksheets](#)
- [Khan Academy](#)
- [Robert Kaplinsky - Great Lesson starters with real world math](#)
- [Dan Meyer Three Act Math - link to spreadsheet](#)

- [Open Middle - challenging problems worth solving](#)
- [Enriching Mathematical Experiences](#)
- [Inside Mathematics - performance assessment tasks](#)
- [Estimation 180: Number sense through engaging daily estimation](#)
- [Illustrative Mathematics](#)
- [Mathematics Assessment Project - lessons, tasks, tests](#)
- [Great Minds](#)
- [AIR Released Items](#)

ESSENTIAL UNDERSTANDINGS

COMPONENTS

ASSESSMENT FOR LEARNING

What is necessary for students to know and be able to do?

Apply and extend an understanding to the system of rational numbers.

- Analyze proportional relationships and use them to solve real-world and mathematical problems
- Represent and analyze quantitative relationships between dependent and independent variables

What are the necessary components of high-quality instruction? How do we get students there?

- [Fluency](#)
- Growing student vocabulary and the language of Mathematics ([Quizlet](#))
- Hands on activities (manipulatives)
- Collaboration
- Use of technology ([Desmos graphing calculator](#))
- Real-life connections: ([POW/POM](#))

How do we know when students have it?

- Assessments - formative & summative ([iReady](#), [Edulastic](#), [Quizlet](#), [Quizizz](#))
- Observation
- Application/rubrics
- Student dialogue - use of mathematical language
- Performance assessment tasks ([Inside Mathematics](#), [Mathematics Assessment Project](#))

GRADE BAND: 6-12

What is necessary for students to know and be able to do? Continued...

- Use functions to model relationships between quantities
- Solve real-world and mathematical problems involving surface area and volume
- Develop an understanding of statistical variability, distributions, and draw inferences
- Rewrite expressions in equivalent forms using algebraic properties to make different characteristics or features visible
- Find solutions to an equation, an inequality, or system of equations
- Use the structure of an equation or inequality to determine an efficient strategy to find the solution, if one exists, and to justify the solution
- Compute areas and volumes of figures by determining how the figure might be obtained from simple figures
- Apply geometric transformations to figures
- Provide an understandable argument that a mathematical procedure is always true, or will always work

What are the necessary components of high-quality instruction? How do we get students there? Continued...

- Stations ([Free Math Worksheets](#))
- Small group
- Differentiation & Intervention
- [Math Talks](#)
- Writing about math ([Yummy Math](#))
- Instructional strategies
- Instructional Videos
- Modeling
- Marvelous mistakes ([My Favorite No](#))
- Error analysis
- Pre-assessment ([Edulastic](#), [Socrative](#))
- Discovery ([Dan Meyer](#), [Robert Kaplinsky](#))
- Immediate feedback ([Edulastic](#), [Khan Academy](#))

What do we do for students who have it?

- [Providing Enrichment activities](#) ([Khan Academy](#), [Yummy Math](#), [Open Middle](#), [Illustrative Math](#))
- Recognition
- Small Group
- Plus (+) standards ([nrich math](#))
- Elective and career tech courses at High School

What do we do for students who don't have it?

- Reteach ([Khan Academy](#))
- Alternative strategies ([Estimation 180](#), [Mathematics Assessment Project](#))
- Chunking the assignments
- Small group
- Online resources
- After School Math Support

Mathematics Framework Team

A special thank you to all of the members of the Mathematics Framework team:

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